Post-doctoral position
Stochastic calculus with the multistable Lévy motion and applications in finance

1 Context

Multistable processes are processes which are, at each time t, tangent to a stable process with index α, but where α varies with t. Such processes have a rich local structure, since both the pointwise Hölder regularity and the intensity of jumps may be tuned independently at each point. They are believed to be good candidates to model various phenomena, in particular in finance.

The simplest multistable process is maybe the symmetric multistable Lévy motion. It is obtained from the so-called symmetric multistable Lévy field, which is defined as follows over $[0,1] \times U$, where U is a closed interval of $\mathbf{R}$ [?]: let $\alpha : [0,1] \to [c,d]$ where either $[c,d] \subset (0,1)$ or $[c,d] \subset (1,2)$ be twice continuously differentiable. Let $(\Gamma_i)_{i\geq 1}$ be a sequence of arrival times of a Poisson process with unit arrival time, $(V_i)_{i\geq 1}$ be a sequence of i.i.d. random variables with distribution $\hat{m}(dx) = dx$ on $[0,1]$, and $(\gamma_i)_{i\geq 1}$ be a sequence of i.i.d. random variables with distribution $P(\gamma_i = 1) = P(\gamma_i = -1) = 1/2$. Assume finally that the three sequences $(\Gamma_i)_{i\geq 1}$, $(V_i)_{i\geq 1}$, and $(\gamma_i)_{i\geq 1}$ are independent and define

$$X(u,v) = C_{\alpha(v)}^{1/\alpha(v)} \sum_{i=1}^{\infty} \gamma_i \Gamma_i^{-1/\alpha(v)} \mathbf{1}_{[0,u]}(V_i).$$

The associated symmetric multistable Lévy motion is by definition:

$$Y(t) = X(t,t).$$

The field and motion above have another representation in terms of sums over a Poisson measure [?]: let Π be a Poisson process on $[0,1] \times \mathbf{R}$ with mean measure $\mathcal{L}[0,1] \times \mathcal{L}(\mathbf{R})$ (where $\mathcal{L}$ is the Lebesgue measure). We will use (X, Y) to denote a random point in $[0,1] \times \mathbf{R}$ of the Poisson process Π. The symmetric multistable Lévy field is equal in law to:

$$X(u,v) = \sum_{(\mathsf{X},\mathsf{Y})\in\Pi} \mathbf{1}_{[0,u]}(\mathsf{X}) \mathsf{Y}^{<-1/\alpha(v)>}$$

where $a^{<b>} = \mathrm{sign}(a)|a|^b$, and where the sum above is to be interpreted as:

$$X(u,v) = \lim_{n\to\infty} \sum_{(\mathsf{X},\mathsf{Y})\in\Pi\cap R_n} \mathbf{1}_{[0,u]}(\mathsf{X}) \mathsf{Y}^{<-1/\alpha(v)>},$$

when $\alpha(v) \geq 1$, with R_n being the rectangle $\{(x,y) : 0 \leq x \leq 1, |y| \leq n\} \subset \mathbf{R}^2$ (the series above converges almost surely).

The symmetric multistable Lévy motion is tangent, at each time t, to the usual Lévy motion $L_{\alpha(t)}$ with index $\alpha(t)$. This means that:

$$\lim_{r \to 0} \frac{Y(u+rt) - Y(u)}{r^h} = L_{\alpha(t)}(t),$$

where the limit is in finite dimensional ditributions.

In addition, for each fixed t, the pointwise Hölder exponent of Y at t is almost surely equal to $\frac{1}{\alpha(t)}$.

2 Subject of the Post-Doctoral Work

The aim of the post-doctoral position will be to study some properties of the stochastic calculus with respect to the multistable Lévy motion in view of applications in finance. Since this process is a semi-martingale, the classical Ito theory applies. Also, specially for applications in finance, it will be interesting to extend the definition above to deal with the non-symmetric case. This raises various difficulties. One major aim of the work will be to compare multistable Lévy motions with different α functions, as well as some of their variants, for computing long-term Value at Risk.

3 Skills and Profile

A strong background in stochastic analysis as well as in financial mathematics is required. Moderate skills in programming would be a plus.

4 Contact

jacques.levy-vehel@inria.fr

5 Place of work

Ecole Centrale Paris, Chatenay Malabry

References

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[2] Le Guével, R. and Lévy Véhel, J. (2009). *A Ferguson - Klass - LePage series representation of multistable multifractional motions and related processes*, preprint. Available at http://arxiv.org/abs/0906.5042.

[3] Samorodnitsky, G. and Taqqu, M.S. (1994). *Stable Non-Gaussian Random Processes*, Chapman and Hall.