# Fractal Analysis of the Motions of a Ball in a Computer Soccer Game

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This paper describes a fractal analysis of the motions of a ball in a computer soccer game. It is important to know the behaviors of the ball in the game. We obtained the 2-dimensional coordinates of the ball by using video processing techniques, and we calculated the values of regularization dimension and box-counting dimension of the time series thus obtained. The values of regularization dimension and box-counting dimension corresponding to the components parallel to the touch line are slightly less than those of the components parallel to the goal line. Also, in fact, these values are rather close to the value of 1, which means that the motions are rather smooth. We also calculated the fractal dimension of the trajectories of the ball on the playing field.

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## I. INTRODUCTION

The motions of players and a ball in a soccer match seem to be quite interesting to both coaches/trainers/players, on the one hand, and spectators on the other. If we know the trajectories of all the players and the ball, we can calculate the relevant interesting behaviors of the dynamics involved during the game. Here, we will focus only on the fractal behaviors of the motions.

In practice, we could not easily obtain the positions of all the players and the ball in a real-world match, due to some technical difficulties. Notice that the motions in a computer soccer game seem to be rather similar to those of a real-world soccer match. So, we decided to try to get the positions from a computer soccer game, which is rather easier to do than from the real-world case. Anyway, this is the first attempt to apply fractal analysis to the behaviors of a ball in a soccer game.

We obtained the coordinates of the ball on a twodimensional playing field by using standard video processing techniques, and we calculated the values of regularization dimension and box-counting dimension of the time series for each component. The values of regularization dimension and box-counting dimension corresponding to the components parallel to the touch line are slightly less than those for the components parallel to the goal line. Also, in fact, these actual values are rather close to the value of 1, which means that the motions are rather smooth. We also calculated the fractal dimension of the trajectories of the ball on the playing field. This fact also means that the trajectories are rather smooth. In Section II, we briefly summarize some basic background knowledge of fractal analysis. Among many measures, the regularization dimension and box-counting dimension will be discussed. In the following section, the properties of the data obtained and the calculated results will be presented. In the final section, some discussions and conclusions are given.

### II. BASIC THEORY OF FRACTAL ANALYSIS

In recent years, the science of fractals has grown into a vast area, encompassing almost all branches of science and engineering [1,2]. Fractals may be found in nature, or are generated in a computer by using an iterative algorithm. The properties of fractals are characterized by:

- 1. Self-similarity; a small portion of a fractal object looks similar to the whole object
- 2. Scaling relationship;
- 3. Nonintegral dimension, which gives a quantitative measure of self-similarity and the scaling law of the fractal object.

To characterize the fractal behavior of the system, we will here use the regularization dimension and box-counting dimension. In this case, we employed FracLab [3] as one of our analysis tools. In FracLab, plenty of helpful methods are well implemented.

Regularization dimension is already well summarized elsewhere [3], so we will omit this here. Instead, we will briefly summarize the definition and properties of

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box-counting dimension [4]. The box-counting dimension (sometimes also called the capacity dimension) provides a relatively simple and appealing way of assigning a fractal dimension to a set S. Let us assume that the set S lies in an N-dimensional Cartesian space. Now, let us cover the set S by a grid of N-dimensional boxes of edge length  $\epsilon$ . We then count the minimum number  $N(\epsilon)$  of boxes needed to cover the set S. We will do this process for successively smaller  $\epsilon$  values. As we let the size  $\epsilon$  of each box get smaller, we expect  $N(\epsilon)$  to increase, because we may need a larger number of smaller boxes to cover all points of the set. The box-counting dimension  $D_B$  is then defined by

$$D_B = \lim_{\epsilon \to 0} \frac{\ln N(\epsilon)}{\ln(1/\epsilon)}.$$
 (1)

This method, however, becomes very time consuming and so becomes impractical in 3- or more dimensional systems, because an enormous number of points are required in order to make sure that a given area in the space is empty and seldom visited. Therefore, it is not so useful for dimension determination in a higher-dimensional system. Further, this method does not keep track of any inhomogeneities in the set, because a given box is counted at most once and only once, no matter how many times the orbit visits the set. In spite of this, the box-counting dimension is widely used in practice for estimating the fractal dimension of a variety of fractal objects.

#### III. RESULTS

It is very difficult, though not impossible, to obtain data from a real-world soccer match. There may be some possible methods for measuring the positions of players and a ball during the game. First, we may implant a strip of radio-frequency (RF) communication chips on the body of each player and the ball. Then, we can collect the signals from the RF chips. If we collect them in at least two different fixed locations, we can get the 2-dimensional positions of the objects. This is a rather direct approach. In this approach, we may get quite accurate positions, but it may interfere with the minds of some players because they are conscious of others measurements. Furthermore, the equipment may easily break down in the vigorous activities between players during the game, such as body collisions or bodily obstructions. So, collecting the data may stop. Otherwise, we must replace any device immediately after breakdown to continue the measurement. The measured data may be rather accurate, but the process costs a lot of money. Second, we can take a rather indirect or passive approach: we record the scenes by using video camcorders, again in at least two different fixed locations. We cannot use just one pair, because the resolution of a camcorder is still too low to identify the locations of a ball played on

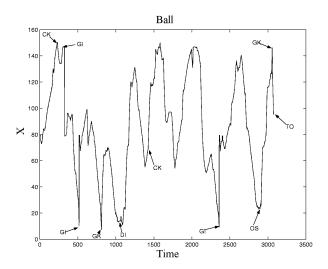


Fig. 1. Time series of x-component of a ball. In the figure, GK, GI, DI, CK, OS and TO represent goal kick, goal-in, draw-in, corner kick, offside and time out, respectively.

the ground. So, we must use several pairs of camcorders to cover the whole playing field. By dividing the playing field into several non-overlapping blocks, each respective pair of camcorders records the scenes from just one block of the field. Thus, we must employ at least 2 pairs of camcorders to obtain the positions of objects for the whole ground. So, in this case there may be some coincidence problems in adjusting the recorded frames relative to each other. This makes the accuracy of the measurement rather poor. This method requires very complicated video processing techniques such as mosaicking, 2-D reconstruction and so on. Also, in this case, the process is costly and not easy to implement.

These methods may also involve huge technical problems, so that we have to circumvent the difficulties. The computer soccer game may be a good alternative. In fact, when we play the electronic soccer game on a computer, we feel that it seems to be nearly statistically similar to a real-world soccer match. This fact makes the analysis of a computer game meaningful. Furthermore the analysis methods we have developed here can be applied both to the virtual game and to the real-world game. Anyway, we could get the data on positions of a ball from a computer soccer match, but this is, however, quite far from a trivial job. The electronic soccer game we used is FIFA WorldCup  $2002^{TM}$  by Electronic Arts (EA) Sports. In this game, if we set the RADAR, at the bottom of the window we have a small inset figure which shows the positions of all the players and the ball. We captured these small inset pictures and made them into a video file for analysis. With this file as an input, we measured the locations of the ball by using image/video processing techniques.

We chose a coordinate system such that the x-axis is along the touchline, and the y-component is parallel to the goal line. The time series of ball motions are shown

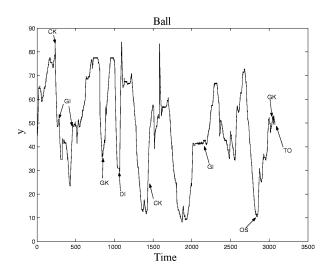


Fig. 2. Time series of y-component of a ball. In the figure, GK, GI, DI, CK, OS and TO represent goal kick, goal-in, draw-in, corner kick, offside and time out, respectively.

Table 1. Basic properties of the data obtained.

Property	Value
Total number of frames	5979
Frame size	$153 \times 85$ pixels
Total play time	398s
Frames per second	15

in Figs. 1 and 2. The total time steps cover 5979 frames. The characteristics of the data are summarized in Table 1. Let us denote the two teams as S and F. At time step t=1, team S kicks off the game. At time t=232, team S gets corner kick. At time t = 325, team F scores a goal and kicks off. At t = 812, team S scores a goal and kicks off again. At t = 1581, team S gets a corner kick. At t = 2358, team F scores second goal and kicks off again. At t = 2908, team F gets an offside foul. At t = 3055, team F gets a goal kick. At t = 3075, the first half is over and the second half starts. Figs. 1 and 2 only show the first half of the game. At t = 3698, team F scores third goal and kicks off again. At t = 4012, team F scores fourth goal and kicks off again. At t = 4450, team S gets a free kick. At t = 4730, team F gets a corner kick. At t = 4878, team S gets a goal kick. At t = 5437, team S has a free kick. At t = 5872, team F gets a corner kick. At t = 5961, team S gets a goal kick. In Figs. 1 and 2, we can see many singular behaviors, such as step-like and cusp-like features. Some of them are due to such events as corner kick, goal kick, kick-off and free kick, or due to some unknown artifacts related to the video processing. The others are intrinsic properties of the system itself. Here, we are focusing on the latter behavior. To see the fractal behaviors of the ball motion, we calculated the regularization dimension  $D_R$ . The calculated results are summarized in Table 2. All the relevant

Table 2. Calculated results for the time series.

Time series	Regularization Dim	Box-counting Dim
$full\_x$	1.12	1.33
$full\_y$	1.17	1.45
$1st\_half\_x$	1.10	1.13
$1st\_half\_y$	1.16	1.18
$2nd\_half\_x$	1.11	1.20
$2nd\_half\_y$	1.17	1.22
$clip 07\_x$	1.05	1.05
clip07_y	1.12	1.18
$clip11\_x$	1.05	1.11
$clip11\_y$	1.10	1.16

quantities are calculated using the FracLab matlab toolbox [3]. The value of x-coordinates of ball position for the full series  $full_x$  is 1.12, and the corresponding values for y-coordinates full\_y is 1.17. The corresponding boxcounting dimensions are 1.33 and 1.45, respectively. The results for 1st\_half are for the 1st half of the game, with time steps ranging from 1 to 3073. Those for 2nd\_half correspond to the 2nd half of the game, which includes time steps from 3074 to 5979. Thus, time series full corresponds to the sum of 1st\_half and 2nd\_half. clip07 corresponds to the longest clip in the 1st half, with time steps from 1581 to 2357, and clip11 corresponds to the longest clip in the 2nd half, with time steps from 3074 to 3697. Thus, these clip07 and clip11 time series are continuous in themselves, but 1st\_half, 2nd\_half and full time series have discontinuities between clips of ball motions. This discontinuity is inevitable, since it is inherent in acquiring the data. For example, if a team scores a goal in a certain clip, then the ball is located in the goal area, and then the next step starts with a kick-off in the center circle area.

Note that a straight line in the time series graph gives  $D_R = 1$ , and that a complicated line completely covering a plane in the time series graph gives  $D_R = 2$ . As can be seen in Table 3, the values are rather close to 1, which corresponds to the value for a single straight line. Thus, we can say that the ball nearly follows a straight line. Notice that y-component values of the regularization dimension are also small, but consistently slightly larger than the corresponding x-component values. This means that the motions along the touch line (x-) direction are somewhat more regular than those along the goal line (y-)direction. Also note that the same holds for the box-counting dimension. The value of box-counting dimension is the upper boundary of the corresponding regularization dimension, and the values corresponding to the 1st\_half, 2nd\_half, full are larger than those of the clip07 and clip11. This is due to the fact that the discontinuities in the 1st\_half, 2nd\_half, full data increase the complexity in the graph, as expected.

We plotted the 2-D trajectory of a ball during  $clip\theta$ ?

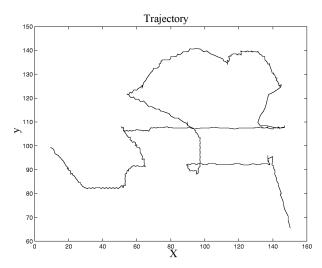


Fig. 3. Trajectory of ball motion during *clip07*. Team S has a corner kick and team F finally scores a goal.

Table 3. Calculational results of the fractal dimension of the 2-D trajectories of the ball.

Data set	Fractal Dim
full	1.61
1st_half	1.54
$2nd\_half$	1.50
clip07	1.05
clip11	1.16

in Fig. 3. To see the 2-D trajectory behaviors of the ball, we calculated its fractal dimensions. The calculated results are shown in Table 3. This shows that the compound trajectories of 1st\_half, 2nd\_half, full data are clearly distinct from the simple continuous trajectory of clip07, clip11. The calculated values for the compound trajectories are comparable to the values for the Brownian random motion, which has the fractal dimension value of 1.5. The values for the clip07 and clip11 are roughly comparable with the values of the corresponding regularization dimension.

## IV. DISCUSSION AND CONCLUSIONS

The calculated data we used were not, in fact, obtained from a real-world soccer match on a soccer pitch. Our data, however, are simulated ones. In the computer soccer game, a human controls some activities of the players in the game and thus interferes in the movements of the players involved in the game. Originally, the movements are controlled as programmed in the computer. These human controls or interferences mainly concentrate on the players around the ball, and, on the other hand, the other players of the game are mainly controlled by the computer program itself.

In conclusion, the component motions of the ball in the computer soccer game are rather smooth. They show a strong persistent long-term memory. The values of regularization dimension and box-counting dimension corresponding to the components parallel to the touch line are slightly less than those for the components parallel to the goal line. Also, in fact, these actual values are rather close to the value 1, which means that the motions are rather smooth. We also calculated the fractal dimension of the trajectories of the ball on the playing field.

In this work, we only considered a game played by computer vs. computer. Needless to say, it is worth extending to various situations such as computer vs. one or two human players, or one human player vs. another human player, etc. In fact, a human player can have a wide range of skills to handle. So, it is also interesting to try to see these effects systematically. The fractal dimension thus obtained can be used to characterize the skewness of the ball motion during the game.

We are preparing a paper [5] on the correlational behaviors between a ball and a player and between two players. This technique can be used to find dominant players and to divide them into closely related subgroups.

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